Status of Eikonal Two-Loop Calculations with Massive Quarks

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Abstract

We present results for two-loop diagrams with massive quarks in the eikonal approximation. Explicit expressions are given for the UV poles in dimensional regularization of several of the required integrals.

1 Introduction

The calculation of threshold corrections to hard scattering cross sections beyond leading logarithms requires the calculation of loop diagrams in the eikonal approximation [1]. One-loop calculations have been performed for all $2 \to 2$ partonic processes in heavy quark [2] and jet [3] production. The soft anomalous dimension matrix Γ_S at one-loop allows the resummation of soft-gluon corrections at next-to-leading logarithm (NLL) accuracy [2]. The exponentiation follows from the renormalization group evolution of Γ_S and involves the calculation of the ultraviolet (UV) poles in dimensional regularization of one-loop diagrams with eikonal lines. To extend resummation to next-to-next-to-leading logarithms (NNLL) two-loop calculations are required. For massless quark-antiquark scattering the two-loop Γ_S was completed in [4]. For heavy quark production, however, the result is not known. In this contribution we present several results for two-loop diagrams involved in the calculation of the two-loop Γ_S for massive quarks. In the eikonal approximation the usual Feynman rules are simplified by letting the

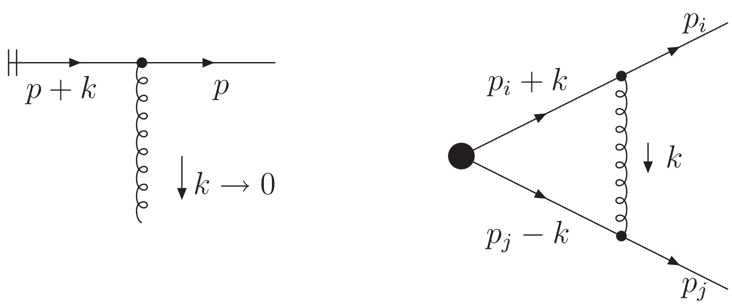

Figure 1: The eikonal approximation (left) and a one-loop diagram (right).

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gluon momentum approach zero (left diagram in Fig. 1):

$$\bar{u}(p)\,(-ig_s T_F^c)\,\gamma^\mu \frac{i(\not{p}+\not{k}+m)}{(p+k)^2-m^2+i\epsilon} \to \bar{u}(p)\,g_s T_F^c\,\gamma^\mu \frac{\not{p}+m}{2p\cdot k+i\epsilon} = \bar{u}(p)\,g_s T_F^c\,\frac{v^\mu}{v\cdot k+i\epsilon}$$

with $p \propto v$, and T_F^c the generators of SU(3).

2 One-loop and two-loop diagrams

We perform our calculation for eikonal massive quarks in Feynman gauge using dimensional regularization with $n = 4-\epsilon$.

We begin with the one-loop diagram in Fig. 1. The momentum integral is given by

$$I_{1l} = g_s^2 \int \frac{d^n k}{(2\pi)^n} \frac{(-i)g^{\mu\nu}}{k^2} \frac{v_i^\mu}{v_i\cdot k} \frac{(-v_j^\nu)}{(-v_j\cdot k)}\,.$$

Using Feynman parametrization, followed by integration over k, and after several manipulations, we find

$$\begin{aligned} I_{1l} &= \frac{\alpha_s}{\pi}\,(-1)^{-1-\epsilon/2}\,2^{5\epsilon/2}\,\pi^{\epsilon/2}\,\Gamma\left(1+\frac{\epsilon}{2}\right)(1+\beta^2)\int_0^1 dx\,x^{-1+\epsilon}(1-x)^{-1-\epsilon} \\ &\quad \times \left\{\int_0^1 dz\left[4z\beta^2(1-z)+1-\beta^2\right]^{-1} - \frac{\epsilon}{2}\int_0^1 dz \frac{\ln\left[4z\beta^2(1-z)+1-\beta^2\right]}{4z\beta^2(1-z)+1-\beta^2} + \mathcal{O}\left(\epsilon^2\right)\right\} \end{aligned}$$

where $\beta = \sqrt{1-4m^2/s}$. The integral over x contains both UV and infrared (IR) singularities. We isolate the UV singularities, $\int_0^1 dx\,x^{-1+\epsilon}\,(1-x)^{-1-\epsilon} = \frac{1}{\epsilon} + \mathrm{IR}$, and find the UV pole and constant terms at one loop:

$$\begin{aligned} I_{1l}^{UV} &= \frac{\alpha_s}{\pi}\frac{(1+\beta^2)}{2\beta}\left\{\frac{1}{\epsilon}\ln\left(\frac{1-\beta}{1+\beta}\right) + \frac{1}{2}\left(4\ln 2+\ln\pi-\gamma_E-i\pi\right)\ln\left(\frac{1-\beta}{1+\beta}\right)\right. \\ &\quad \left. + \frac{1}{4}\ln^2(1+\beta) - \frac{1}{4}\ln^2(1-\beta) - \frac{1}{2}\mathrm{Li}_2\left(\frac{1+\beta}{2}\right) + \frac{1}{2}\mathrm{Li}_2\left(\frac{1-\beta}{2}\right)\right\}. \end{aligned}$$

More details on this one-loop integral are given in [5]. We now continue with the two-loop diagrams (these are the eikonal versions of the diagrams involved in the calculation of the two-loop heavy quark form factor [6]). In Fig. 2, we show a diagram with two gluons exchanged between the massive quarks (left) and the crossed diagram (right). We denote by I_1 the integral for the first diagram and by I_2 that for the crossed diagram. We have

$$I_1 = g_s^4 \int \frac{d^n k_1}{(2\pi)^n} \frac{d^n k_2}{(2\pi)^n} \frac{(-i)g^{\mu\nu}}{k_1^2} \frac{(-i)g^{\rho\sigma}}{k_2^2} \frac{v_i^\mu}{v_i\cdot k_1} \frac{v_i^\rho}{v_i\cdot(k_1+k_2)} \frac{(-v_j^\nu)}{-v_j\cdot k_1} \frac{(-v_j^\sigma)}{-v_j\cdot(k_1+k_2)}\,.$$

We note that I_1 is symmetric under $k_1 \leftrightarrow k_2$ as is the integral for the crossed diagram, I_2. Utilizing the properties of these two integrals and the one-loop integral, I_{1l}, we find the relation

$$I_1 = \frac{1}{2}(I_{1l})^2 - I_2\,.$$

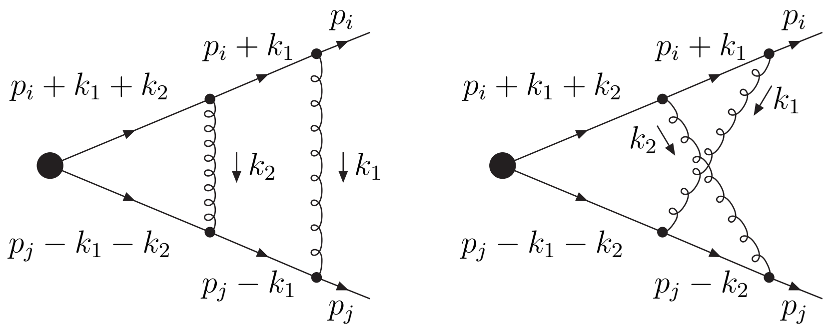

Figure 2: Two loop diagrams with two-gluon exchanges.

Therefore I_1 is determined once we calculate I_2. For the crossed diagram, we have

$$I_2 = g_s^4 \int \frac{d^n k_1}{(2\pi)^n} \frac{d^n k_2}{(2\pi)^n} \frac{(-i)g^{\mu\nu}}{k_1^2} \frac{(-i)g^{\rho\sigma}}{k_2^2} \frac{v_i^\mu}{v_i \cdot k_1} \frac{v_i^\rho}{v_i \cdot (k_1+k_2)} \frac{(-v_j^\nu)}{-v_j \cdot (k_1+k_2)} \frac{(-v_j^\sigma)}{-v_j \cdot k_2} \, .$$

We begin with the k_2 integral and after some work find

$$\begin{aligned} I_2 &= -i\,\frac{\alpha_s^2}{\pi^2}\, 2^{-4+\epsilon}\, \pi^{-2+3\epsilon/2}\, \Gamma\left(1-\frac{\epsilon}{2}\right) \Gamma(1+\epsilon)\,(1+\beta^2)^2 \int_0^1 dz \\ &\times \int_0^1 \frac{dy\,(1-y)^{-\epsilon}}{\left[2\beta^2(1-y)^2 z^2 - 2\beta^2(1-y)z - \frac{(1-\beta^2)}{2}\right]^{1-\epsilon/2}} \int \frac{d^n k_1}{k_1^2\, v_i \cdot k_1\, \left[((v_i - v_j)z + v_j)\cdot k_1\right]^{1+\epsilon}} \, . \end{aligned}$$

Now we proceed with the k_1 integral and separate the UV and IR poles. After many steps, we find the $1/\epsilon^2$ and $1/\epsilon$ UV poles of I_2:

$$\begin{aligned} I_2^{UV} &= -\frac{\alpha_s^2}{\pi^2}\frac{(1+\beta^2)^2}{8\beta^2}\frac{1}{\epsilon}\left\{ \ln\left(\frac{1-\beta}{1+\beta}\right)\left[2\,\mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right) + 4\,\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)\right.\right. \\ &\qquad \left. + 2\,\mathrm{Li}_2\left(\frac{-(1-\beta)}{1+\beta}\right) - \ln(1+\beta)\,\ln(1-\beta) - \zeta_2\right] \\ &\qquad \left. - 2\ln^2\left(\frac{1-\beta}{1+\beta}\right)\ln\left(\frac{1+\beta}{2\beta}\right) + \frac{1}{3}\ln^3(1-\beta) - \frac{1}{3}\ln^3(1+\beta) - \mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right) + \zeta_3 \right\} . \end{aligned}$$

We now proceed with the diagrams in Fig. 3 that involve internal quark and gluon loops. For the quark loop we find

$$I_{ql} = (-1) n_f g_s^4 \int \frac{d^n k}{(2\pi)^n} \frac{d^n l}{(2\pi)^n} \frac{v_i^\mu}{v_i \cdot k} \frac{(-v_j^\rho)}{(-v_j \cdot k)} \frac{(-i)g^{\mu\nu}}{k^2} \frac{(-i)g^{\rho\sigma}}{k^2} \mathrm{Tr}\left[-i\gamma^\nu \frac{i\not{l}}{l^2}(-i)\gamma^\sigma i\frac{(\not{l}-\not{k})}{(l-k)^2}\right] .$$

After many steps (see also [5]) we extract the UV poles

$$\begin{aligned} I_{ql}^{UV} &= -n_f \frac{\alpha_s^2}{\pi^2}\frac{(1+\beta^2)}{6\beta}\left\{ \frac{1}{\epsilon^2}\ln\left(\frac{1-\beta}{1+\beta}\right) + \frac{1}{\epsilon}\left[-\mathrm{Li}_2\left(\frac{1+\beta}{2}\right) + \mathrm{Li}_2\left(\frac{1-\beta}{2}\right)\right.\right. \\ &\qquad \left.\left. + \frac{1}{2}\ln^2(1+\beta) - \frac{1}{2}\ln^2(1-\beta) + \left(\frac{5}{6} + 4\ln 2 + \ln\pi - \gamma_E - i\pi\right)\ln\left(\frac{1-\beta}{1+\beta}\right)\right]\right\} . \end{aligned}$$

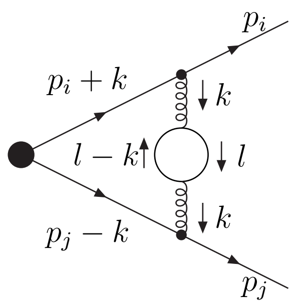

Figure 3: Two-loop diagrams with quark and gluon loops.

For the gluon-loop integral, we have

$$
\begin{aligned}
I_{gl} &= \frac{1}{2} g_s^4 \int \frac{d^n k}{(2\pi)^n} \frac{d^n l}{(2\pi)^n} \frac{v_i^\mu}{v_i \cdot k} \frac{(-v_j^\nu)}{(-v_j \cdot k)} \frac{(-i) g^{\mu\mu'}}{k^2} \frac{(-i) g^{\rho\rho'}}{l^2} \frac{(-i) g^{\sigma\sigma'}}{(k-l)^2} \frac{(-i) g^{\nu\nu'}}{k^2} \\
&\times \left[g^{\mu'\rho} (k+l)^\sigma + g^{\rho\sigma} (k-2l)^{\mu'} + g^{\sigma\mu'} (-2k+l)^\rho \right] \\
&\times \left[g^{\rho'\nu'} (l+k)^{\sigma'} + g^{\nu'\sigma'} (-2k+l)^{\rho'} + g^{\sigma'\rho'} (k-2l)^{\nu'} \right] .
\end{aligned}
$$

We calculate the UV poles and find

$$
\begin{aligned}
I_{gl}^{UV} &= -\frac{19}{96} \frac{\alpha_s^2}{\pi^2} \frac{(1+\beta^2)}{\beta} \left\{ \frac{1}{\epsilon^2} \ln \left(\frac{1-\beta}{1+\beta} \right) + \frac{1}{\epsilon} \left[-\mathrm{Li}_2 \left(\frac{1+\beta}{2} \right) + \mathrm{Li}_2 \left(\frac{1-\beta}{2} \right) \right. \right. \\
&\quad \left. \left. + \frac{1}{2} \ln^2 (1+\beta) - \frac{1}{2} \ln^2 (1-\beta) + \left(\frac{58}{57} + 4 \ln 2 + \ln \pi - \gamma_E - i\pi \right) \ln \left(\frac{1-\beta}{1+\beta} \right) \right] \right\} .
\end{aligned}
$$

We also have to add a diagram to those in Fig. 3 involving a ghost loop. The corresponding integral is

$$
I_{gh} = (-1) g_s^4 \int \frac{d^n k}{(2\pi)^n} \frac{d^n l}{(2\pi)^n} \frac{v_i^\mu}{v_i \cdot k} \frac{(-v_j^\rho)}{(-v_j \cdot k)} \frac{i}{l^2} l^\nu \frac{i}{(l-k)^2} (l-k)^\sigma \frac{(-i) g^{\mu\nu}}{k^2} \frac{(-i) g^{\rho\sigma}}{k^2}
$$

and a calculation of its UV poles gives

$$
\begin{aligned}
I_{gh}^{UV} &= -\frac{\alpha_s^2}{\pi^2} \frac{(1+\beta^2)}{96\beta} \left\{ \frac{1}{\epsilon^2} \ln \left(\frac{1-\beta}{1+\beta} \right) + \frac{1}{\epsilon} \left[-\mathrm{Li}_2 \left(\frac{1+\beta}{2} \right) + \mathrm{Li}_2 \left(\frac{1-\beta}{2} \right) \right. \right. \\
&\quad \left. \left. + \frac{1}{2} \ln^2 (1+\beta) - \frac{1}{2} \ln^2 (1-\beta) + \left(\frac{4}{3} + 4 \ln 2 + \ln \pi - \gamma_E - i\pi \right) \ln \left(\frac{1-\beta}{1+\beta} \right) \right] \right\} .
\end{aligned}
$$

We also note that the integral for another diagram involving an internal gluon loop with a four-gluon vertex vanishes.

There are additional diagrams not discussed here, also including self-energies and counterterms. The color factors for all diagrams have been calculated and must be accounted for in the final result.

Acknowledgements

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